Introduction

Migration algorithms are often formulated as the adjoint of a linear forward modelling operator rather than the inverse. This is because the adjoint operators are cost effective and robust. This means that adjoint operators tolerate imperfections in the data and do not demand that the data provide full information (Claerbout, 1995). It is well known that most of the operators used for seismic processing are non-unitary (Claerbout, 1992). As a result, migration operators undo the time and phase shifts of the modelling operator but do not account for amplitude (Claerbout, 1995). In other words, reflectivity obtained from applying migration operator on a seismic data needs amplitude correction. Equations (1) and (2) show how seismic data and migrated image are related to reflectivity model and seismic data respectively.

$$d = Lm \,, \tag{1}$$

$$m_{mig} = L^T d \ . \tag{2}$$

Where **d** represents the scattered seismic data, **m** is the reflectivity model, $\mathbf{m_{mig}}$ is the migration image, and **L** is the forward modelling operator associated with a specific survey geometry, source wavelet, and velocity-density model. The migration operator is the adjoint of the forward modelling operator.

Since the migration operator is not unitary; causes the migrated images having distorted amplitudes. Also, lateral focusing and defocusing generated by illumination problems due to finite-recording aperture and lateral velocity variations, can bias amplitudes in typical migration results. In addition to amplitude correction, benefits of least squares migration (LSM) are reduction in migration artifacts and an increase in spatial resolution (Nemeth, et al. 1999). LSM finds a reflectivity model $\mathbf{m_{LSM}}$ that minimizes the 2-norm of the difference between actual seismic data and data obtained from applying the modelling operator on $\mathbf{m_{LSM}}$.

$$m_{LSM} = \arg\min_{m}(\|Lm - d\|_2^2) \ . \tag{3}$$

LSM due to the nature of the 2-norm has a tendency to smooth the resulting image. Since the reflectivity distribution is sparse and discontinuous, by using a proper regularization in the cost function, one can attenuate artifacts and obtain a non-smooth image.

The well known F-K and the phase-shift migration methods are strictly valid only within the homogeneous models and layered models, respectively. In this paper, to extend the F-K domain methods to laterally inhomogeneous media, a generalized F-K migration operator (Pai, 1988) and its adjoint are used in order to perform migration and modelling operations. In the proposed method, the downward continuation is accomplished, not using plane waves individually as in the F-K or in the phase-shift method, but by employing the whole spectrum of plane waves simultaneously. In the following section the generalized F-K method is discussed. Then sparse regularized LSM is proposed and finally the results and conclusions are presented.

Methodology

First of all we start from wave equation and extend the F-K migration method to laterally variant velocity fields.

Generalized F-K Migration Method

The method is a generalization of the F-K and the phase-shift methods, valid in arbitrarily varying models. As mentioned above, in this method whole spectrum of plane waves are simultaneously used in order to downward continue the wave filed. Starting from 2-D wave equation, applying the Fourier transform over time axis gives:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{v^2(x,z)}\right)\psi_\omega(x,z)=0\ . \tag{4}$$

Since velocity has a functionality of **x**, it should be noted that the Fourier transform of equation (4) with respect to **x** requires convolving the Fourier transform of the wave field and the Fourier transform of the velocity dependent term. In addition to taking all the plane waves in to account, we use matrix representation of the 2-D Fourier domain wave equation:

$$\left(\frac{\partial^2}{\partial z^2}+\begin{pmatrix}-k_1^2 & & 0\\ & \ddots & \\ 0 & & -k_N^2\end{pmatrix}+\omega^2\begin{pmatrix}s_1(z) & \dots & s_N(z)\\ \vdots & \ddots & \vdots\\ s_N(z) & \cdots & s_1(z)\end{pmatrix}\right)\psi_{\omega,k_x}(z)=0\ , \tag{5}$$

$$K_x=\begin{pmatrix}-k_1^2 & & 0\\ & \ddots & \\ 0 & & -k_N^2\end{pmatrix},\quad S(z)=\begin{pmatrix}s_1(z) & \dots & s_N(z)\\ \vdots & \ddots & \vdots\\ s_N(z) & \cdots & s_1(z)\end{pmatrix}. \tag{6}$$

In the above equations $\mathbf{s_i}$'s are the Fourier transform components of the square of slowness at depth **z**. after reordering some terms of the above equation we get:

$$\left(\frac{\partial^2}{\partial z^2}+K_z^2\right)\psi_{\omega,k_x}(z)=-\omega^2\left[S(z)-S_{diag}(z)\right]\psi_{\omega,k_x}(z)\ , \tag{7}$$

where

$$S_{diag}(z)=\begin{pmatrix}s_1(z) & & 0\\ & \ddots & \\ 0 & & s_1(z)\end{pmatrix},\ K_z^2=K_x+\omega^2S_{diag}(z)=\begin{pmatrix}-k_1^2+\omega^2s_1(z) & & 0\\ & \ddots & \\ 0 & & -k_N^2++\omega^2s_1(z)\end{pmatrix}=\begin{pmatrix}\kappa_1^2 & & 0\\ & \ddots & \\ 0 & & \kappa_N^2\end{pmatrix}. \tag{8}$$

Using definitions in equation (8), the one-way downward wave equation can be obtained as:

$$P(z_{n+1},z_n)=\begin{pmatrix}e^{-j\kappa_1(z_{n+1}-z_n)} & & 0\\ & \ddots & \\ 0 & & e^{-j\kappa_N(z_{n+1}-z_n)}\end{pmatrix},\quad G(z_{n+1},z_n)=\frac{-1}{2j}\begin{pmatrix}e^{-j\kappa_1(z_{n+1}-z_n)}/\kappa_1 & & 0\\ & \ddots & \\ 0 & & e^{-j\kappa_N(z_{n+1}-z_n)}/\kappa_N\end{pmatrix}. \tag{9}$$

The downward continuation equation and the algorithm for obtaining the downward continued image are presented below. Further information about generalized F-K migration can be found in Pai (1988).

$$\psi_{\omega,k_x}(z_{n+1})=P(z_{n+1},z_n)u(z_n)-\omega^2\left[S(z_n)-S_{diag}(z_n)\right]G(z_{n+1},z_n)u(z_n)(z_{n+1}-z_n)\ . \tag{10}$$

In equation (10), $u(z_n)$ is the upcoming wave at depth level z_n. Steps required for downward continuation using generalized F-K migration method is proposed below:

1) Start with zero-offset seismic data recorded at the earth's surface.
2) Take the Fourier transform of seismic data with respect to **t** and **x**.
3) For each depth step, calculate the downward continued version for all frequency components. Then sum all the downward continued frequency components.
4) Take the inverse Fourier transform of the the result with respect to **x**.

Sparsity Promoting Least Squares Migration

As mentioned in the introduction, for robustness, typical migration algorithms use the adjoint of modelling operator to perform migration. Usually the migration operator is not unitary. So the migrated image is a distorted version of the reflectivity model. Based on the equations (1) and (2), it could be concluded that the distortion function is Hessian matrix as in equation (11).

$$m_{mig} = (L^T L)m \ . \tag{11}$$

LSM removes the effect of Hessian matrix by finding a reflectivity model that minimises the 2-norm of difference between zero-offset data and data obtained by applying modelling operator on the reflectivity model.

$$m_{LSM} = \arg\min_m (\|Lm - d\|_2^2) \ . \tag{12}$$

Since reflectivity model is non-smooth and sparse, by imposing a sparsity constraint on the reflectivity model one can reach a model with higher resolution and less artifacts. The LSM due to the nature of the 2-norm, tries to distribute the error on all components and smoothes the resulting image. Therefore we add a regularization term to equation (12) in order to make the final image non-smooth. The 1-norm is used to achieve a sparse structure for the reflectivity.

$$m_{LSM} = \arg\min_m (\|Lm - d\|_2^2 + \lambda \|m\|_1) \ . \tag{13}$$

By solving the above optimization problem using Basis Pursuit Denoising (BPDN) (van den Berg and Friedlander, 2008) we obtain a sparse reflectivity model that fits the zero-offset seismic data.

Examples

This method is tested on a synthetic reflectivity model shown in figure 1. There are 100 traces, separated by 1 meter. The wavelet width is four time divisions, each time division being 0.003 seconds, with a total of 100 time divisions shown. The reflectivity amplitude is one on the reflectors and zeros everywhere. The velocity model for the reflectivity model is shown in figure 2. A synthetic data has been obtained by applying the modelling operator that discussed above, on the synthetic reflectivity model.

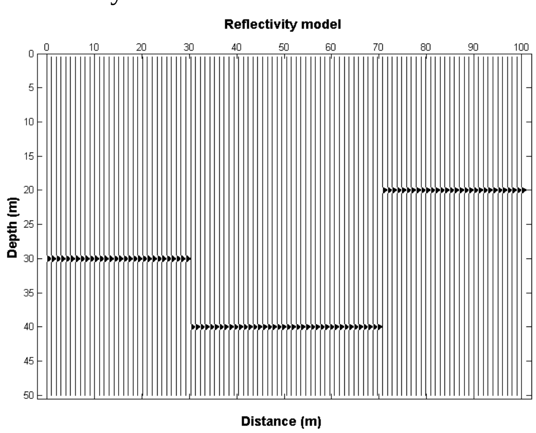

Figure 1 *The Synthetic reflectivity model.*

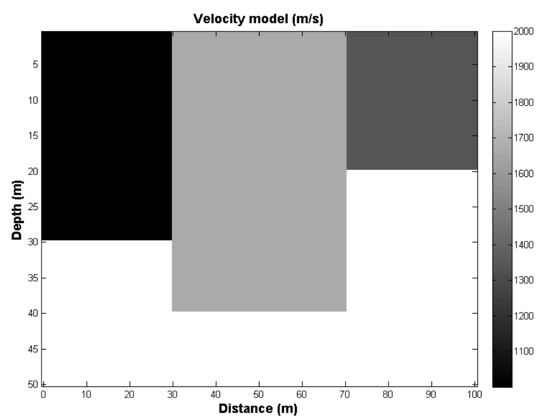

Figure 2 *The Synthetic velocity model.*

As it is seen, the velocity model has strong lateral variations. Figure 3 shows the obtained synthetic data, in other words, the result of applying the modelling operator on reflectivity model. Figure 4 is the result of applying the generalized F-K migration operator on the synthetic data. Figure 5 illustrates the result of applying the LSM method on the synthetic data.

By comparing figure 4 and figure 5, it can be concluded that both methods had correctly positioned the reflectors but the LSM algorithm has managed to obtain an image with much higher resolution. In addition, amplitude distortion has been taken into account by LSM in contrast to generalized F-K migration. There are artifacts in figure 4 but in figure 5 the amplitudes is non-zero around the reflectors and zero everywhere.

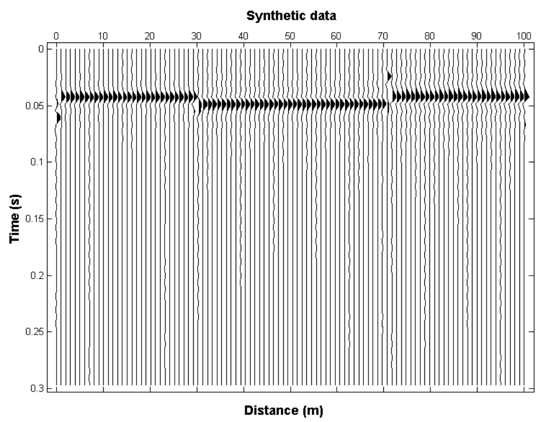

Figure 3 *Synthetic data.*

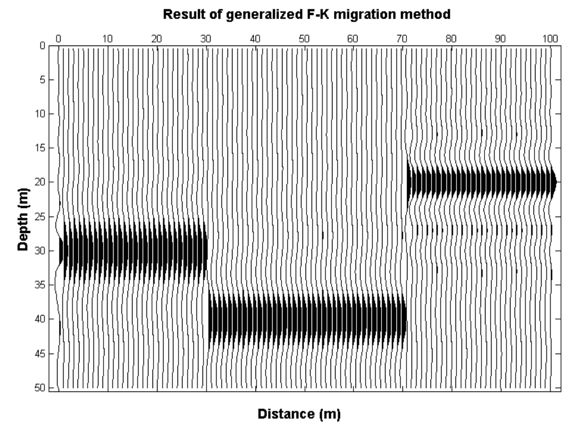

Figure 4 *The result of applying the generalized F-K migration on figure 3.*

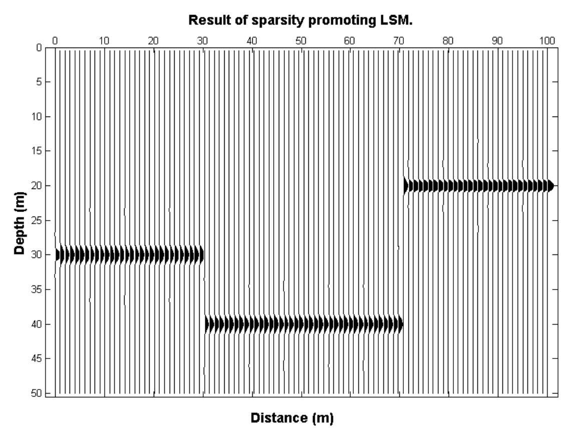

Figure 5 *Sparsity Promoting LSM.*

Conclusion

In this paper we proposed a sparsity based migration using the generalized F-K migration method that uses:

1) All the plane waves together in order to compensate for laterally velocity variations.
2) 1-norm based least squares migration to obtain sparse and non-smooth reflectivity model.

The efficiency of the method was tested on synthetic seismic data.

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